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A LARGE STRAIN MULTISCALE APPROACH USING C^1 CONTINUITY FINITE ELEMENT

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Abstract — A second order two scale computational homogenization approach for modeling deformation responses of heterogeneous material at large strain is proposed. The structure at macrolevel is discretized by the C^1 continuity displacement based triangular finite elements derived in this contribution, while the standard C^0 quadrilateral finite element is used for the discretization at microscale. All algorithms derived have been implemented into the finite element software ABAQUS. The performance of the proposed multiscale approach is demonstrated by an example.

Key-words — Multiscale computational approach, Second-order computational homogenization, Large strain formulation, C^1 continuity finite element.

1 Introduction

During recent years, a special attention has been directed to the investigation of the relations between the macroscopic material behaviour and its microstructure, and various numerical multiscale techniques have been developed. However, in more recent formulations the two scale computational homogenization approach has mostly been used [1-4]. The computational homogenization method does not require an explicit *a priori* constitutive relation at macrolevel which allows modeling of complex microstructure geometry as well as deformation responses. In the case when the macroscopic stress-strain behavior depends on the characteristic size of the microstructure, and higher order effects such as strain localization phenomena and material softening are present, second-order homogenization based on a nonlocal strain gradient continuum theory should be considered. Herein the second-order stress and strain are included. However, the second order homogenization requires C^1 continuity at the numerical solution of the boundary value problem at macrolevel, which implicates the requirement that both displacements and strains must be continuous functions.

In this contribution, according to the two scale computational homogenization strategy, the macroscopic second-order formulation based on the large strain theory and C^1 continuity finite element discretization are used, while the boundary value problem at microscopic scale is considered on a statistically representative sample of material, named Representative Volume Element (RVE). In this framework, the microscopic problem formulation obeys the classical continuum theory implemented in the C^0 finite element formulation, as usually used in available literature [1, 2, 5]. The C^1 continuity displacement based finite element formulation is proposed, where displacement field as only independent variable is considered as an advantage in comparison to other formulations employing interpolations of both displacements and displacement gradients. The derived finite element together with the two-scale computational procedure is implemented into the FE program Abaqus using user subroutines [5, 6]. The efficiency and accuracy of the proposed multiscale computational approach are demonstrated by an example.

2 Micro-macro algorithm

The micro-macro algorithm consists of two different levels. The first level represents the macromodel, discretized by the C^1 continuity triangular finite elements. As the second level, the microstructure is presented by the RVE, here discretized by the C^0 quadrilateral four-node finite elements. In every macrolevel integration point of the structural mesh, RVE microanalysis is performed. The macrolevel deformation gradient \mathbf{F}_M and its gradient \mathbf{G}_M are transformed into the RVE boundary nodal displacements using periodic boundary conditions. After solving RVE boundary value problem, the first Piola-Kirchhoff stress tensor \mathbf{P} and the double stress tensor \mathbf{Q} , which energy conjugate to \mathbf{F} and \mathbf{G} , respectively, and the constitutive matrices \mathbf{C}_M are obtained by a homogenization procedure. The computational algorithm proposed is performed in the frame of the finite element program ABAQUS [6] by simultaneous combination of the micro- and macroanalyses and using user subroutines, as it is summarized in the flowchart in Figure 1. To enable nonlinear computations, the incremental values are transformed from macro to microlevel. Analogous procedure under small strain assumption is described in [5].

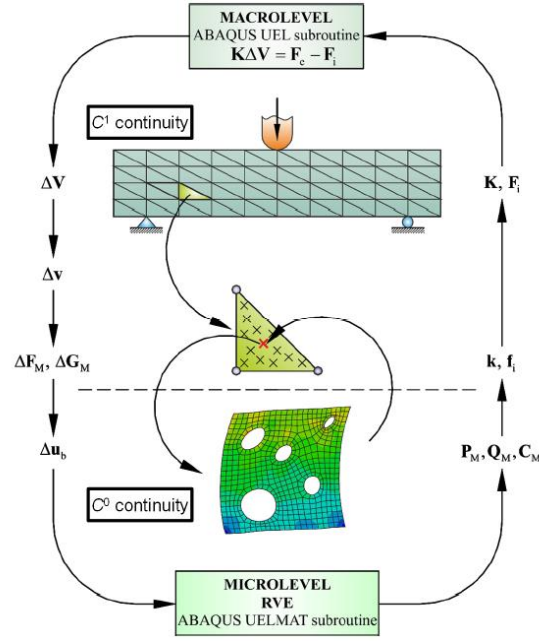


FIG. 1 Micro-macro computational scheme

For the solution of the RVE boundary value problem, the boundary conditions are expressed by the displacements which are related to the macroscopic kinematic values \mathbf{F}_M and \mathbf{G}_M according to the relation

$$\mathbf{u}_m = \mathbf{X}^T (\mathbf{F}_M - \mathbf{I}) + \frac{1}{2} \mathbf{X}^T \mathbf{G}_M \mathbf{X} + \mathbf{r}. \quad (1)$$

Herein \mathbf{X} is a spatial coordinate on the RVE boundary, and \mathbf{r} represents the microfluctuation field [5]. Employing the well-known Hill-Mandel energy condition, and after the mathematical transformations which are analogous to those described in [1, 5], the constitutive tangent operators \mathbf{C}_{PF} , \mathbf{C}_{PG} , \mathbf{C}_{QF} , \mathbf{C}_{QG} are derived for the computation of element stiffness matrix at the macrolevel.

3. C^1 large strain finite element formulation

The plane strain two dimensional triangular finite element is proposed. The finite element has three nodes and 36 degrees of freedom with the displacement field approximated by the full fifth order polynomial. The nodal degrees of freedom are the two displacement components u_1, u_2 and their first and second order derivatives with respect to the Cartesian coordinates, as shown in Figure 2.

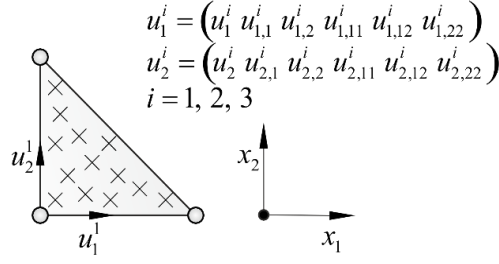


FIG. 2 C^1 triangular finite element

The element stiffness relation may be written in standard matrix form $\mathbf{K} \Delta \mathbf{v} = \mathbf{F}_e - \mathbf{F}_i$ with $\Delta \mathbf{v}$ as nodal displacement increment and \mathbf{K} as the element stiffness matrix consisting of the following parts.

$$\mathbf{K} = \mathbf{K}_{PF} + \mathbf{K}_{PG} + \mathbf{K}_{QF} + \mathbf{K}_{QG}, \quad (3)$$

where the particular matrices are defined as

$$\begin{aligned} \mathbf{K}_{PF} &= \int_{A_0} \mathbf{B}_F^T \mathbf{C}_{PF} \mathbf{B}_F dA_0, \quad \mathbf{K}_{PG} = \int_{A_0} \mathbf{B}_F^T \mathbf{C}_{PG} \mathbf{B}_G dA_0, \\ \mathbf{K}_{QF} &= \int_{A_0} \mathbf{B}_G^T \mathbf{C}_{QF} \mathbf{B}_F dA_0, \quad \mathbf{K}_{QG} = \int_{A_0} \mathbf{B}_G^T \mathbf{C}_{QG} \mathbf{B}_G dA_0. \end{aligned} \quad (4)$$

The values \mathbf{F}_e and \mathbf{F}_i on the right hand-side of the stiffness equation are the external and internal nodal force vectors

$$\mathbf{F}_e = \int_{s_0} (\mathbf{N}^T \mathbf{t} + \mathbf{B}_G^T \boldsymbol{\tau} \mathbf{n}) ds_0, \quad \mathbf{F}_i = \int_{A_0} (\mathbf{B}_F^T \mathbf{P} + \mathbf{B}_G^T \mathbf{Q}) dA_0.$$

\mathbf{B}_F and \mathbf{B}_G are the matrices containing the first and second partial derivatives of the interpolation functions \mathbf{N} , respectively. The values \mathbf{t} and $\boldsymbol{\tau}$ denote the traction and double traction vectors while \mathbf{n} is the vector of unit outward normal to the boundary.

4 Numerical example

A rectangular strip of the length and height of 0.2 m with the thickness of 1 m subjected to bending is considered. The deformation response of a square model discretized by the 16 plane strain triangular finite elements is studied, as shown in Figure 3a. The loading and boundary conditions are imposed, yielding the deformed configuration with constant curvature. The material data are the Young's modulus $E = 210 \text{ GPa}$, the Poisson's ratio $\nu = 0.3$, the yield stress of 250 MPa and the constant tangent modulus of 250 MPa describing linear isotropic hardening. The deformation responses are considered using both the small strain and the large strain second-order two-scale computational approach. The two RVE side length of 0.2 mm with different porosity of 13% and 27% randomly distributed voids are analyzed, as presented in Figures 4a and 4b. For the sake of comparison, the homogeneous material is also considered. The RVE discretization has been performed using the quadrilateral finite elements available in ABAQUS.

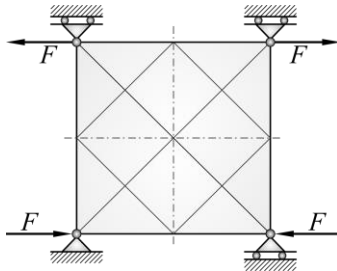
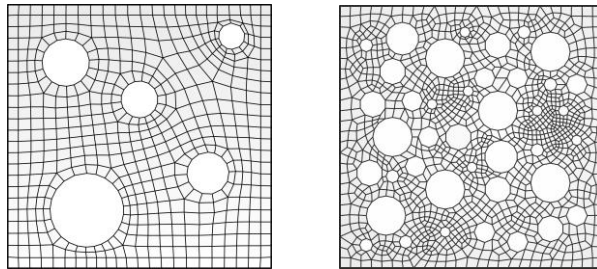


FIG. 3 Bending problem



a)

b)

FIG. 4 RVE-s with a) 13% voids, b) 27% voids

The moment-curvature diagram is shown in Figure 5. The diagram displays that the stiffness of the analyzed specimens is significantly decreased when the porosity is increased from 13% to 27 % in both the small strain and the large strain regimes, as expected. Accordingly, for the same curvature the bending moment is significantly reduced if the porosity is increased. It is also evident that the bending moment at a certain curvature is reduced if the large strain computation is performed in the nonlinear structural behavior.

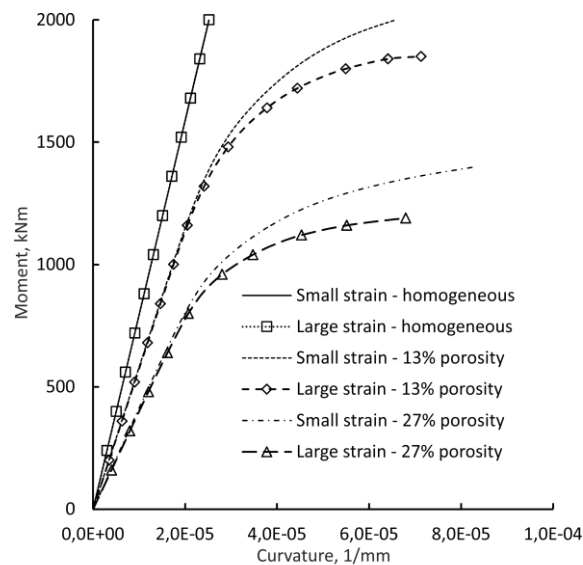


FIG. 5 Moment-curvature diagram

5. Conclusion

A micro-macro computational strategy employing the second order computational homogenization scheme for heterogeneous materials at large strain has been presented. The C^1 continuity large strain triangular finite element is derived and implemented into the FE software ABAQUS. Other algorithms proposed are also embedded into ABAQUS via user subroutines. It is to note that the use of the ABAQUS subroutines significantly contributes to the numerical efficiency of the time demanding numerical computation. The numerical example demonstrates the efficiency and the accuracy of the proposed multiscale formulation.

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